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The Eddy Dissipation Turbulence Energy Cascade Model

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ABSTRACT

The turbulence energy cascade model used in the Eddy Dissipation Concept for combusting flow is presented and discussed in relation to existing knowledge of relevant turbulent flows. The cascade consists of a stepwise model for energy transfer from larger to smaller scales and for energy dissipation from each scale level by viscous forces. The cascade model makes a connection between the viscous fine structures, where combustion takes place, and the larger transporting eddies which are simulated by turbulence models. Thus, fine-structure quantities are expressed in terms of turbulence energy and dissipation. The model is compared to turbulence-energy-spectrum data for the inertial subrange and the dissipative range for nonreacting and reacting flows. The model is also discussed in relation to isotropic decaying turbulence in the transition from initial to final periods of decay. It is concluded that the energy cascade model captures important features of the turbulence structural interaction and dissipation.

1 INTRODUCTION

In the Eddy Dissipation Concept for combusting flows, Magnussen used a turbulence energy cascade model (Magnussen and Hjertager, 1976; Magnussen *et al.*, 1978; Magnussen, 1981a, 1981b, 1989; Byggstøyl and Magnussen, 1983). Some issues relating to this cascade model will be presented and discussed in the following. The objective is to give relevant background and clarify some issues in the Eddy Dissipation Concept.

Magnussen's model has been used in several commercial and research codes for combustion flow calculations. However, for various reasons, the background for the model has not been presented and discussed in the literature. This paper seeks to answer many inquiries about the considerations behind the Eddy Dissipation Concept.

The purpose of a model is to depict the features of a phenomenon that are expected to be important, without pretending to represent the full phenomenon in all aspects. On the other hand, a model – in particular an engineering model – has to be applicable within certain limitations of human, material and financial resources. We think the Eddy Dissipation cascade model that is outlined and discussed here is such a link between science and technology. The model has been in widespread use for combustion predictions for many years. In this study, the model has been compared to data for the turbulence energy spectrum. Recent data for the dissipative range of nonreacting and reacting flows enable the model to be assessed.

Averaged over time, the scales of turbulence are continuously distributed over a wide spectrum. Mechanical energy is transferred from the mean flow to large eddies and then further to smaller and smaller eddies (Richardson, 1922). The larger eddies carry the major part of the kinetic energy. Smaller eddies whirl faster but contain less energy. The smallest eddies have the highest frequency and the largest viscous stresses. Viscous friction transfers mechanical energy to heat. This dissipation occurs at all scale levels but is largest in the smaller eddies. The spectrum of larger and smaller eddies is often considered as an energy or turbulence cascade.

Onsager (1945, 1949) seems to be the first to suggest a stepwise cascade model for the turbulence spectra. Each step was represented by a frequency or wave number that was twice the value of the preceding step. A model of this type was presented by Tennekes and Lumley (1972) (see also Lumley, 1992; Lumley *et al.*, 1996) for the inertial, or nonviscous, part of the spectrum, and they formulated the model in terms of the wave number.

Combustion takes place where reactant molecules are mixed, that is, mainly in the smaller scales. In these fine structures, the local molecular flux is the largest. Similarly, the local viscous stress, and thus the dissipation, is the largest in the fine structures. In combusting flows, the heat release produces some mechanical energy, and this affects the turbulence spectrum. Unfortunately – as seen from a combustion viewpoint – most cascade models and other spectrum models are restricted to the nonviscous part of the spectrum.

In this paper the turbulence cascade model will be compared with existing models and experimental data for the turbulence energy spectrum; for the inertial range as well as the viscous (dissipative) range.

The energy transfer in the cascade model can also be regarded as a dissipation model. Such models are assessed by comparison with theory and experimental data for decaying isotropic turbulence for high and low Reynolds numbers. This is done in Section 4.

In a turbulent flow, the large-scale eddies transfer mechanical energy to smaller eddies. Spatial transport by turbulent motions is dominated by the large eddies. Thus, the energy-transfer model will be compared to turbulence viscosity models and experimental data for boundary-layer flows.

2 CASCADE MODEL

2.1 Input and Output of the Model

The input to the cascade model presented here is the turbulence energy and a turbulence time-scale or length-scale variable, e.g. the dissipation rate of turbulence energy. These can be found from modeled transport or balance equations (that is, a statistical turbulence model). With some modifications, the cascade model can also be used in large-eddy simulations, or with transport equations on different steps in the cascade. The output of the cascade model is the mean rate of molecular mixing in a turbulent flow. The mean reaction rate is assumed to be a linear function of this quantity.

2.2 Outline of the Model

FIGURE 1 Magnussen (1981a) presented a model with a characteristic frequency or strain rate. Figure 1 illustrates the model for the transfer of mechanical energy from the mean flow, through turbulence energy, to heat. w' is the feed of mechanical energy from the mean flow to turbulence. For steady-state turbulence, this is the production of turbulence kinetic energy. The sum of $q' + q'' + \ldots + q^*$ is the dissipation rate of turbulence kinetic energy, ε. When the turbulence quantities, such as turbulence energy, are found from transport equations, w' represents the total supply of turbulence kinetic energy.

The first level in the turbulence structure is the large, energy-rich eddies. It is characterized by a velocity scale u' (which is taken equal to $(\frac{2}{3}k)^{1/2}$, where k is the turbulence energy) and a length scale L'. A frequency or strain rate can be expressed as

$$\omega' = u'/L'. \tag{1}$$

These quantities vary in space and time and are determined by the turbulence model, e.g. a k-ε model or a Reynolds-stress-equation model. This level represents the whole spectrum because it contains the effect of smaller scales. The next level represents the part of the spectrum where the characteristic frequency is $\omega'' = 2\omega'$, velocity u'', and length L''. In the same way as the first, this level was assumed to contain the effect of all subsequent levels. Likewise, the n-th level was characterized by $\omega_n = 2\omega_{n-1}$, u_n, and L_n. In the smallest eddies, ω^*, u^*, and L^* are of the same order of magnitude as the Kolmogorov scales.

The transfer from the first to the second level, w'', is equal to the sum of the dissipation from all subsequent levels, thus the dissipation is $\varepsilon = q' + w''$.

Production, which feeds mechanical energy to the first level, is a product of a turbulence stress and a mean-flow strain rate. The feed to the second level was modeled according to the same

pattern:

$$w'' = \tfrac{3}{2} C_{\mathrm{D}1} 2 u''^2 \omega'. \tag{2}$$

The transfer from the first level to the second was modeled to be proportional to the square of a characteristic fluctuation at the second level and proportional to the characteristic frequency or strain rate at the first level. This is an analogy to production, which is proportional to the square of a characteristic fluctuation at the first level, and also proportional to the strain rate at the mean-flow level. In this expression, $2\omega'$ can be replaced by ω''.

The transfer of mechanical energy to thermal energy from the first level was modeled as

$$q' = C_{\mathrm{D}2} \nu \omega'^2. \tag{3}$$

This is the direct dissipation from the first level. This model is an analogy to the dissipation term in the mechanical energy equation, which is proportional to the viscosity and the square of a strain rate.

In the same way, expressions can be developed for w''' and q'', and so on further down in the cascade. It was assumed that the transfer from one level to the next follows the same model for all levels. This gave a set of equations for each level. For the n-th level,

$$w_n = \tfrac{3}{2} C_{\mathrm{D}1} \omega_n u_n^2 \tag{4}$$

and

$$q_n = C_{\mathrm{D}2} \nu \omega_n^2, \tag{5}$$

and the balance, when using a quasi-steady assumption, is $w_n = q_n + w_{n+1}$. The final step $(n = *)$ is the fine structures. Here, $w^* = \frac{3}{2} C_{\mathrm{D}1} \omega^* u^{*\,2}$ and $q^* = C_{\mathrm{D}2} \nu \omega^{*\,2}$, and w^* is equal to q^*.

For simplicity it was assumed that the numerical factors $C_{\mathrm{D}1}$ in the expressions above were the same, and a constant; and similarly for $C_{\mathrm{D}2}$.

For high and moderate Reynolds numbers, the dissipation is small at the upper levels. That is, for a small n, q_n is negligible compared to w_n, and w_n is approximately equal to w_{n+1}. Then, for $n = 2$, u'''^2 approximates $\frac{1}{2} u''^2$. If, in the same way, it is assumed that $u''^2 \approx \frac{1}{2} u'^2$, then $w'' = C_{\mathrm{D}1} \omega' \frac{3}{2} u'^2$. Further, the turbulence energy, $k = \frac{3}{2} u'^2$, can be introduced. The transfer of mechanical energy from the first turbulence level was approximated to be

$$w'' = C_{\mathrm{D}1} \omega' k. \tag{6}$$

In the turbulent-combustion model (Magnussen, 1981a,1989) the cascade model was used with $C_{\mathrm{D}1} = 0.135$ and $C_{\mathrm{D}2} = 0.5$. Since a model is only an approximation, numerical values for the constants have to be a best fit with respect to several types of flows. In a turbulence model, all constants are more or less linked together. Thus, the constants in the dissipation model depend on constants in other parts of the complete model.

The value of $C_{\mathrm{D}1} = 0.135$ was chosen by using the approximation $\varepsilon \approx w'' = \frac{3}{2} C_{\mathrm{D}1} u'^3 / L'$. The turbulence viscosity $\nu_{\mathrm{t}} = u' \cdot L'$ can be expressed by $\nu_{\mathrm{t}} = \frac{3}{2} C_{\mathrm{D}1} u'^4 / \varepsilon = \frac{2}{3} C_{\mathrm{D}1} k^2 / \varepsilon$. Then, $\frac{2}{3} C_{\mathrm{D}1}$ corresponds to the constant C_μ $(= 0.09)$ in the widely used k-ε model (Launder and Spalding, 1974).

2.3 Energy Cascade and Fine Structures

A turbulence model gives expression to phenomena that are connected to the first level of the energy cascade. Chemical reactions take place where the reactants are mixed molecularly, that is, mainly in the fine structures, which are the small eddies. The energy-cascade model outlined above gives a relation between large and small scales.

The production of turbulence kinetic energy, or transfer of energy from the mean flow to turbulence, is a function of quantities that are characteristic of the mean flow and of the turbulence. Likewise, the transfer from the first to second levels has to be a function of quantities that are connected to the two levels.

In the combustion model, the energy cascade was used to find a set of relations that control the combustion rate, see e.g. Magnussen (1981a,1989) or Byggstøyl and Magnussen (1983). Here, the expressions will be redeveloped without introducing values to the constants.

At a high Reynolds number, the transfer from the first to the second level, w'', is far greater than the direct dissipation from the first level, q'. That is, the dissipation ε is approximately equal to the energy transfer w'', and thus

$$\varepsilon = \tfrac{3}{2} C_{\mathrm{D1}} \frac{u'^{\,3}}{L'}. \tag{7}$$

From the model expressions above, it can be found that $\varepsilon = \frac{4}{3} q^*$, thus

$$\varepsilon = \tfrac{4}{3} C_{\mathrm{D2}} \nu \frac{u^{*\,2}}{L^{*\,2}}, \tag{8}$$

and with a balance for the last level,

$$\varepsilon = \tfrac{4}{3} w^* = 2 C_{\mathrm{D1}} \frac{u^{*\,3}}{L^*}. \tag{9}$$

The two last equations give the characteristic scales for the fine structures,

$$L^* = \frac{2}{3} \left(\frac{3 C_{\mathrm{D2}}^3}{C_{\mathrm{D1}}^2} \right)^{1/4} \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}, \tag{10}$$

$$u^* = \left(\frac{C_{\mathrm{D2}}}{3 C_{\mathrm{D1}}^2} \right)^{1/4} (\nu \varepsilon)^{1/4}. \tag{11}$$

These scales are of the same order of magnitude as the Kolmogorov scales.

The ratio of the fine-structure mass to the total mass was expressed (Magnussen *et al.*, 1978; Magnussen, 1981a) as

$$\gamma^* = \left(\frac{u^*}{u'} \right)^3 = \left(\frac{3 C_{\mathrm{D2}}}{4 C_{\mathrm{D1}}^2} \right)^{3/4} \left(\frac{\nu \varepsilon}{k^2} \right)^{3/4}. \tag{12}$$

This is the intermittency factor of the fine structures. If the Taylor microscale λ is defined by $\lambda^2 = 10\nu k/\varepsilon$, it follows from Eq. (12) that γ^* is proportional to $R_\lambda^{-3/2}$, where $R_\lambda = u'\lambda/\nu$ is the Taylor microscale Reynolds number. The postulate in Eq. (12) implies, together with Eqs. (7) and (9), that

$$\gamma^* \approx \frac{L^*}{L'}. \tag{13}$$

This is similar to the model by Corrsin (1962). Here, a factor of $\frac{3}{4}$ was left out. It was believed that the fine structures are mainly located in the highly strained regions between larger energy-rich eddies.

The mass transfer between fine structures and surroundings, divided by the fine-structure mass, was modeled (Magnussen *et al.*, 1978; Magnussen 1981a)

$$\dot{m}^* = 2\frac{u^*}{L^*} = \left(\frac{3}{C_{\mathrm{D2}}}\right)^{1/2} \left(\frac{\varepsilon}{\nu}\right)^{1/2}. \tag{14}$$

The inverse of this quantity, $\tau^* = 1/\dot{m}^*$, is regarded as the characteristic time-scale of the fine structures. This is the fluid-dynamics time-scale for the chemical reactions, and is an important quantity for the treatment of chemical kinetics (see Gran and Magnussen, 1996a,b).

The mass transfer between fine structures and surroundings, divided by the total mass, then became

$$\dot{m} = \dot{m}^*\gamma^* = \frac{3}{4C_{\mathrm{D1}}} \left(\frac{12C_{\mathrm{D2}}}{C_{\mathrm{D1}}^2}\right)^{1/4} \left(\frac{\nu\varepsilon}{k^2}\right)^{1/4} \frac{\varepsilon}{k}. \tag{15}$$

This quantity can be interpreted as the mean rate of molecular mixing. The reaction rate for a chemical species was assumed to be a linear function of $\dot{m}$. Thus, the choice of numerical values for the model constants C_{D1} and C_{D2} in the turbulence cascade affects the local combustion-rate predictions. Using this model, the mean chemical reaction rate and the characteristic quantities related to the fine structures are expressed in terms of a normal engineering statistical, or mean-value, turbulence model, such as the k-ε model.

3 THE TURBULENCE ENERGY SPECTRUM

In this section, the Eddy Dissipation cascade model will be related to existing models and experimental data for the turbulence energy spectrum.

3.1 Models and Experimental Data

The distribution of turbulence energy on different scales is expressed in terms of the 3-dimensional energy spectrum, $E(\kappa)$. For high-Reynolds-number flows, Kolmogorov (1941) expressed the energy spectrum for the inertial subrange

$$E(\kappa) = C_K \varepsilon^{2/3} \kappa^{-5/3}, \tag{16}$$

where κ is the wavenumber. The parameter C_K was assumed to be a constant, known as Kolmogorov's constant. In this range, the symbol ε represents the transfer of mechanical energy across a wavenumber, from larger to smaller scales. Experiments indicated that C_K has an approximate value of 1.5 (Sreenivasan, 1995). The question remains how large a "high" Reynolds number is.

At higher wavenumbers (smaller scales), viscous forces become important. In the dissipative range, the Kolmogorov scales can be defined. Compared to the large eddies, this range contains a small portion of the turbulence energy and therefore has a small value of the energy spectrum. However, the dissipation spectrum, $D(\kappa)$, has its maximum value in this range.

The shapes of $E(\kappa)$ and $D(\kappa)$ in the dissipative range cannot be determined as easily as in the inertial subrange. A few researchers have attempted to make a curve-fit to experimental data of $E(\kappa)$. One type of model extends the inertial-subrange spectrum formulation:

$$E(\kappa) = C_K \varepsilon^{2/3} \kappa^{-5/3} \exp[-\alpha(\kappa\eta)^m]. \tag{17}$$

Here, $\eta = (\nu^3/\varepsilon)^{1/4}$ is the Kolmogorov length scale. Smith and Reynolds (1991) found that Eq. (17), with $m = 2$ and $\alpha = (C_K \cdot 1.355)^{3/2}$, gave the best model for laboratory-scale flows. They used $C_K = 1.7$ instead of 1.5, but the profile was not very different for the two values. Other researchers have suggested models with a second term, proportional to κ^{-1}, see Yakhot and Zakharov (1993), She and Jackson (1993), and Sirovich *et al.* (1994). However, these models still contain numerical constants to be quantified in the $E(\kappa)$ formulation.

When analyzing experimental data, She and Jackson (1993) found the peak of the dissipation spectrum at $\kappa \approx 0.1\eta^{-1}$. The model by Smith and Reynolds (1991) gives a dissipation-spectrum peak value at a wavenumber above $0.2\eta^{-1}$. That is, the model by She and Jackson predicts that the energy is dissipated at wavenumbers which are somewhat higher (relative to η^{-1}) than that in the model by Smith and Reynolds. This difference is probably due to the relatively low Reynolds numbers in the experiments.

The experiments and models referred to above were made for flows of homogeneous mixtures and nearly constant temperature and density. In a recent study, Furukawa *et al.* (1996) have measured the energy spectrum for a combusting approach flow. They presented data taken from the intermittent local reaction zone, data taken continuously from the same flow, and data from a nonreacting flow. Compared with the nonreacting flow, the energy spectrum of the reacting flow was increased at the higher wavenumbers. In particular, the turbulence increased in the local reaction zone of the approach flow. The range of wavenumbers and the Kolmogorov scale were about the same in the approach flow and the nonreacting flow. Also in the the local reacting zone, the wavenumbers were in the same range. However, the Kolmogorov scale derived from the local reaction zone was found to be 3–4 times larger than that derived from the continuous data of the same flow and that derived from the nonreacting flow. Since combustion heat release generally increases the kinematic viscosity, this is a reasonable finding, and these effects have to be considered when evaluating a model.

3.2 Cascade Model

The energy spectrum is a continuous function distributed over a wide range of wavenumbers or length scales. A cascade, on the other hand, is a stepwise representation of the same energy distribution. A single value represents a certain wavenumber or length-scale interval. A step in the cascade with a wavenumber κ_n represents the interval (κ_n^-, κ_n^+), or $(\kappa_n^-, \kappa_{n+1}^-)$. The next step has wavenumber κ_{n+1} and represents $(\kappa_{n+1}^-, \kappa_{n+1}^+)$, and so forth.

It has been suggested (*e.g.* Onsager, 1949) that the spectrum can be represented by a cascade where the wavenumber is doubled from step to step. For the inertial subrange, a model of this type has been formulated by Tennekes and Lumley (1972). They used $\kappa_n^- = 0.62\kappa_n$, $\kappa_n^+ = 1.62\kappa_n$, and $\kappa_{n+1} = 2.6\kappa_n$, which actually were approximations of $e^{-1/2}$, $e^{1/2}$, and e^1 times κ_n. This choice gave an interval $\Delta\kappa_n = \kappa_n^+ - \kappa_n^-$ equal to κ_n, and the steps are approximately equal to unity on a logarithmic scale.

Magnussen's Eddy Dissipation cascade model is expressed in steps of a characteristic frequency, or strain rate ω, rather than in wavenumber steps. A relation to the energy spectrum is needed for comparison with experimental data and with other models.

The length scale of a step can be expressed by a wavenumber according to

$$L_n = (b \cdot \kappa_n)^{-1}, \tag{18}$$

where b is a constant. Further, it was assumed that a step represents a wavenumber interval bounded by $\kappa_n^- = a\kappa_n$. The parameters a $(0 < a < 1)$ and b will be discussed later.

Unlike the cascade of Tennekes and Lumley, each step in Magnussen's cascade represents the energy of all the subsequent steps, that is

$$\tfrac{3}{2}u_n^2 = \int_{\kappa_n^-}^{\infty} E(\kappa)d\kappa. \tag{19}$$

Applying this for the next step, and using the energy-transfer model, gave

$$\tfrac{3}{2}u_n^2 - \tfrac{3}{2}u_{n+1}^2 = \tfrac{3}{2}u_n^2\left(1 - \frac{w_{n+1}}{2w_n}\right) = \int_{a\kappa_n}^{a\kappa_{n+1}} E(\kappa)d\kappa. \tag{20}$$

Except for the smallest scales, w_{n+1} equals w_n, so that the parenthesis is equal to $1/2$. With the relation in Eq. (18), the ratio of the wavenumbers is

$$\frac{\kappa_{n+1}}{\kappa_n} = \frac{L_n}{L_{n+1}} = \frac{u_n}{u_{n+1}}\frac{\omega_{n+1}}{\omega_n} = 2\sqrt{2}\left(\frac{w_n}{w_{n+1}}\right)^{1/2}. \tag{21}$$

The parameter a was not defined as a part of the model, but as a means to relate the model to the energy spectrum. It seemed natural to choose a value so that $\kappa_n^+ = a^{-1}\kappa_n$, which implies $a^2 = \kappa_{n-1}/\kappa_n$. By this choice, the steps are centered around κ_n on a logarithmic wavenumber axis. The base of the logarithm is $a^{-2} = \kappa_n/\kappa_{n-1}$, which can be rewritten by $\log_{(a^{-2})}(\kappa/\kappa_n) = \ln(\kappa/\kappa_n)/\ln(a^{-2})$. This turned out very similar to the cascade model of Tennekes and Lumley (see above) for the inertial subrange, that is, for most of the cascade. For this range, $\kappa_n^- = 0.59\kappa_n$ was obtained.

The dissipation occurs mainly in the fine structures. In the Eddy Dissipation cascade model, the dissipation from the last step, $q^* = \frac{3}{4}\varepsilon$, can be related to the dissipation spectrum by

$$q^* = \int_{a\kappa_*}^{\infty} D(\kappa) d\kappa, \tag{22}$$

where $\kappa_* = (bL_*)^{-1}$, cf. Eq. (18).

Within this framework, the Eddy Dissipation cascade model can be compared to models for the energy spectrum.

Inertial subrange With the assumption $w_n = w_{n+1} = \varepsilon$, and with Kolmogorov's model (Eq. 16), Eq. (20) can be written

$$\tfrac{3}{4} u_n'^2 = \tfrac{3}{4} C_K \varepsilon^{2/3} a^{-2/3} \kappa_n^{-2/3}. \tag{23}$$

When the model in Eq. (4) was introduced together with Eq. (18), this gave $b = 2a/(3C_{\mathrm{D1}}C_K^{3/2})$. With $C_{\mathrm{D1}} = 0.135$, $C_K = 1.5$, and $a = 0.59$, this relation gave $b = 1.59$.

Dissipative range The limit for the last level is

$$a\kappa_* = a(bL_*)^{-1} = \frac{3}{2}\left(C_{\mathrm{D1}}^2/3C_{\mathrm{D2}}^3\right)^{1/4} a(b\eta)^{-1}. \tag{24}$$

In the cascade model, the ratio of the energy transfers to the last two levels is $w_{*-1}/w_* = 5/4$, and then from Eq. (21), $a = (\kappa_*/\kappa_{*-1})^{-1/2} = 0.56$. With this value for a, together with $b = 1.59$ from above and the model constants $C_{\mathrm{D1}} = 0.135$ and $C_{\mathrm{D2}} = 0.50$, the limit of the last level is $a\kappa_* = 0.25\eta^{-1}$.

An alternative definition of a, $a_n = (1 + a_{n-1})\,\kappa_{n-1}/\kappa_n$, gave about the same result for the dissipative region.

This outcome of the model can be compared with data and empirical models by integrating the dissipation spectrum. The range representing 3/4 of the dissipation corresponds to the last level of the cascade model. We let $A\eta^{-1}$ be the lower wavenumber limit for this range. Numerical integration of the dissipation spectrum from the model by Smith and Reynolds (Eq. 17) gave $A = 0.198$ for $C_K = 1.5$, and $A = 0.180$ for $C_K = 1.7$. This corresponds to the limit $a\kappa_*$ from the present model.

If the model constant C_{D2} was decided by these data, the result would be a value of about 0.7. As mentioned, some experimental data have shown a lower value of $\kappa\eta$ for the peak of the dissipation spectrum. This may imply a lower value for A, and thus a larger C_{D2}. On the other hand, combustion is likely to increase small-scale turbulence and thus the dissipation at high wavenumbers. This points toward a lower C_{D2}.

4 ISOTROPIC DECAYING TURBULENCE

4.1 Testing a Cascade Model

The energy transfer in the cascade model can be regarded as a dissipation model which can be tested against theory and experimental data for isotropic decaying turbulence. This case

is the simplest type of turbulent flow and has been intensively studied and analyzed by a large number of investigators. The experimental setup is a uniform plug flow behind a grid in a wind tunnel. It can be stated that one-point closure models cannot fully represent this type of flow. For instance, such models do not carry any information about the shape of the energy spectrum. This shape will influence the development of the decaying turbulence far downstream of the grid. A two-point closure is required to represent the shape of the energy spectrum.

However, for engineering purposes, turbulence models are one-point closures, like the well-known two-equation models. Isotropic turbulence is not a "practical flow" but is used as a test case for models due to its simplicity. Industrial flows often have zones of weak and decaying turbulence.

This flow is a test case for turbulence models because all effects other than dissipation can be neglected. In the zone close to the grid, the turbulence Reynolds number is high. This is known as the initial period of decay and is dominated by inertia forces. Far downstream in the flow, the turbulence intensity and the turbulence Reynolds number decrease. In this zone, turbulence is dominated by viscous forces. This is known as the final period of decay.

4.2 Modeling

For isotropic turbulence, the equation for turbulence kinetic energy is simplified to

$$-\frac{dk}{dt} = \varepsilon. \tag{25}$$

Here, k and ε are the turbulence energy and its dissipation rate, and t is time.

In the k-ε model, the viscous effects in decaying turbulence were modeled (Jones and Launder, 1972) by modifying the standard-model ε equation,

$$-\frac{d\varepsilon}{dt} = C_{\varepsilon 2} f_\varepsilon \frac{\varepsilon^2}{k}, \tag{26}$$

where f_ε is a function of the turbulence Reynolds number. At high Reynolds numbers, like the initial period of decay, this function is unity. The constant $C_{\varepsilon 2}$ was chosen from experiments (Comte-Bellot and Corrsin, 1966; Ferziger, 1980) showing that $k \sim t^{-p}$. This gave $C_{\varepsilon 2} = 1.8$ for $p = 1.25$.

The final period of decay occurs when the effect of inertia forces is negligible. Batchelor and Townsend (1948b) determined from a theoretical analysis that $k \sim t^{-q}$, where $q = 5/2$, and their experiments gave the same result. Later measurements by Bennett and Corrsin (1978) and by Wei *et al.* (1988) gave q values from 2.50 to 2.52. This gives $C_{\varepsilon 2} f_\varepsilon = 1.4$ at a low Reynolds number.

In the k-ε model, the function f_ε was designed to fit experimental data. Hanjalić and Launder (1976) suggested

$$C_{\varepsilon 2} f_\varepsilon = 1.8 - 0.4 \exp\left(-\tfrac{1}{36} R_T^2\right). \tag{27}$$

Here, R_T can be expressed $R_T = \frac{3}{20}R_\lambda^2$. Coleman and Mansour (1991) have suggested an alternative formulation based on results from direct numerical simulations,

$$C_{\varepsilon 2} f_\varepsilon = 1.8 - 0.4 \exp(-0.13 R_\lambda). \tag{28}$$

Another way to model flows or zones at a low Reynolds number is to introduce viscous terms in the model in addition to the energy-transfer dissipation term. Such models have previously been suggested by Rotta (1951a,b) and by Chou and Huang (1975). In addition to the k equation, an equation is needed for a second turbulence quantity.

The Eddy Dissipation cascade model provides a dissipation model of this type, with $\varepsilon = w'' + q'$ in the k-equation, and the frequency ω' as the second variable. The model equations can be written

$$-\frac{dk}{dt} = C_{\mathrm{D}1}\omega' k + C_{\mathrm{D}2}\nu\omega'^2 \tag{29}$$

$$-\frac{d\omega'}{dt} = C_{\omega 1}\omega'^2 + C_{\omega 2}\frac{\nu\omega'^3}{k}. \tag{30}$$

In the initial period of decay, the first right-hand-side terms dominate in the two equations. The ratio of the constants is determined from the experiments, $C_{\omega 1} = 0.8 \cdot C_{\mathrm{D}1}$. Similarly, in the final stage of decay, the viscous terms dominate, and the ratio of the constants can be found, $C_{\omega 2} = 0.7 \cdot C_{\mathrm{D}2}$.

4.3 Transition from High to Low Reynolds Numbers

The k-ω model in Eqs. (29) and (30), and the k-ε model Reynolds number function in Eq. (27) was calibrated against experimental data for high and very low Reynolds numbers. However, this does not guarantee that the transition from high to low Reynolds numbers is well modeled.

The different models can be compared by computing the nondimensional decay of the dissipation rate. For k-ε models, it is seen directly from Eq. (26) that

$$\frac{k}{\varepsilon^2}\left(-\frac{d\varepsilon}{dt}\right) = C_{\varepsilon 2} f_\varepsilon, \tag{31}$$

which is only a function of the turbulence Reynolds number.

In the transition period both dissipation terms in the k-equation, Eq. (29), are operative. The ratio of the first to the second dissipation term, w'' to q', is only a function of the Reynolds number R_λ,

$$R_\lambda = \frac{1}{C_{\mathrm{D}1}}\left(\frac{20\, C_{\mathrm{D}2}}{3(1 + w''/q')}\right)^{1/2}\frac{w''}{q'}. \tag{32}$$

For the k-ω model, Eqs. (29) and (30), the nondimensional decay rate can be expressed

$$\frac{k}{\varepsilon^2}\left(-\frac{d\varepsilon}{dt}\right) = \frac{\mathrm{w}''}{\varepsilon} + \frac{C_{\omega 1}}{C_{\mathrm{D}1}}\left(\frac{\mathrm{w}''}{\varepsilon}\right)^2 + \left(2\frac{C_{\omega 1}}{C_{\mathrm{D}1}} + \frac{C_{\omega 2}}{C_{\mathrm{D}2}}\right)\frac{\mathrm{w}''}{\varepsilon}\frac{q'}{\varepsilon} + 2\frac{C_{\omega 2}}{C_{\mathrm{D}2}}\left(\frac{q'}{\varepsilon}\right)^2. \tag{33}$$

This can be calculated as a function of turbulence Reynolds number through Eq. (32). In Figure 2 the present model is compared with Rotta's model, the function of Hanjalić and Launder, Eq. (27), and the function of Coleman and Mansour, Eq. (28). The effect of the viscous forces in the models is shown by the departure from the high Reynolds number value of the decay rate. In Rotta's model, this value is 1.7 and in the other models it is 1.8.

FIGURE 2

As mentioned above, values $C_{D1} = 0.135$, $C_{D2} = 0.5$ were used in the combustion model (Byggstøyl and Magnussen, 1983; Magnussen, 1989). Further, ratios $C_{\omega 1}/C_{D1}$ and $C_{\omega 2}/C_{D2}$ were kept equal to 0.8 and 0.7 respectively, according to the outline above. The figure shows that the effect of viscous forces is weaker in the model of Hanjalić and Launder, acting only at Reynolds numbers less than 10. Rotta's model (1951a) has the strongest effect of the viscous forces.

The tendency in the two-term models, *i.e.* the present model and Rotta's model, is the same as in Coleman and Mansour's function based on direct-numerical-simulation results. The present model has a greater contribution from viscous forces. An increased value of C_{D1} (to let us say 0.2), which will be indicated in the Discussion below (Sec. 5.1), would give a curve closer to the function of Coleman and Mansour.

4.4 Comparison with Experimental Data and Numerical Simulations

The models can be tested against experimental data, if such are available. It seems that no one has carried out measurements systematically from a high to a low Reynolds number. Hanjalić and Launder (1976) and Byggstøyl (1984) compared the models with experimental data by Batchelor and Townsend (1948b). These experiments exhibited the asymptotic behavior characteristic of the final period of decay. Batchelor and Townsend (1947,1948a) also reported experiments for the initial period of decay.

More recent measurements reported by Bennett and Corrsin (1978) and by Wei *et al.* (1988) also showed the end of the transition period, and perhaps the final period. From the experiments referred to here, the transition from the initial to the final period seemed to take place in the region of Reynolds number R_λ from 5 to 15. The precise value depends on the initial conditions of the flow. Direct numerical simulations (Schumann and Patterson, 1978; Mansour and Wray, 1994) of decaying isotropic turbulence support this view.

The data from the literature mentioned above, show that we cannot discern one single value for R_λ, where the two dissipation terms in Eq. (29) are to be equal. The width of the relevant interval is also unclear. However, one can figure out an interval where the transition is likely to occur. The Eddy Dissipation model and the other models discussed here, with the possible exception of Rotta's model, predict transition within this interval.

5 DISCUSSION

5.1 Turbulence Viscosity and Logarithmic Layer

In Section 2.2, the parameter C_{D1} in Eq. (6) has been related to the constant C_μ in the k-ε model. A starting point for deciding the value of C_{D1} is experimental data for the logarithmic

part of a boundary layer. Here, the production of turbulence energy is approximately equal to the dissipation. The production is $P_k = u_\tau^3/\kappa x_2$ and turbulence viscosity is $\nu_t = \kappa x_2 u_\tau$, where u_τ is the wall-friction velocity, x_2 is the distance from the wall, and κ is the von Karman constant. In the logarithmic layer, experimental data (Klebanoff, 1955) showed values for $u_\tau^2/\overline{u_2^2}$ about 1 and for u_τ^2/k about 0.25. However, all the values varied across the logarithmic layer. The constant C_μ in the standard k-ε model (Launder and Spalding, 1974) was determined by $\sqrt{C_\mu} = u_\tau^2/k$ which uses the value of 0.3. This value, combined with values of the other constants in the model, was chosen as a best fit for a number of different flows.

Furthermore, the choice of C_{D1} is influenced by the turbulence-viscosity definition, $\nu_\mathrm{t} = u' \cdot L' = u'^2/\omega'$, where u'^2 is equal to $\frac{2}{3}k$. Possibly, the transverse Reynolds stress $\overline{u_2^2}$ is a more relevant quantity for transverse transport than the turbulence energy. If $\nu_\mathrm{t} = \overline{u_2^2}/\omega'$ were chosen, the experimental data (Klebanoff, 1955) would have given a value for C_{D1} of about 0.20 - 0.30. However, relating the turbulence viscosity to turbulence energy has proved to be reasonable through widespread use of the k-ε and other k-equation models. From the two decades of experience, it also seems that the value 0.09 was a reasonable choice for C_μ.

5.2 Turbulence Energy Spectrum

The review of spectrum data by Sreenivasan (1995) indicated an inertial subrange in flows with Reynolds number R_λ above 50. However, these data were derived from the longitudinal spectra. Only a few sets of transverse spectra indicated a Reynolds number perhaps as high as 1000. Thus, the three-dimensional energy spectrum might have had an inertial subrange only at turbulence Reynolds numbers that were higher than those in most industrial devices. Sreenivasan's review included data from grid-generated turbulence, wall-bounded and free shear layer flows, and atmospheric flows.

Most available spectrum data were measured in nonreacting, nearly isothermal flows. In combustion, large gradients occur in concentrations, temperature and density. Few details are known about how this affects the energy spectrum. The results from one experimental study (Furukawa *et al.*, 1996) indicated that combustion generates turbulence at small scales. This energy is likely to be dissipated at the same or smaller scale levels. Hence, combustion may increase the turbulence energy spectrum and the dissipation spectrum at high wave numbers. This is consistent with the fine-scale combustion model. When exothermic reactions occur in narrow structures – be it sheets, ribbons, or tubes – the fine-scale structures will expand. The surrounding fluid will be moved, and thus, small-scale kinetic energy is generated.

5.3 Dynamics of Cascade Model

The cascade model presented is "dynamic" in the sense that the turbulence energy and the turbulence Reynolds number can be found from transport equations. Thus, the integral of the spectrum varies, as well as the width of the spectrum, according to the output of the turbulence model. This means that the number of cascade steps varies throughout a simulation.

On the other hand, a quasi-steady energy balance was assumed at each level in the cascade model (except at the first level). Spectral cascade models without this assumption can be found in the literature, and can be extended to the viscous dissipation range. However, these

models were developed under conditions of constant density and constant viscosity, and the spectral energy transfer models are still under development. At the time being, they do not seem to be ready to be taken into use for combustion calculations (cf. Sec. 5.2).

5.4 Intermittency and Fine Structures

The smaller scales of high-Reynolds-number turbulence appear very unevenly distributed in space. The volume fraction (or mass fraction) occupied by the structures might be called the intermittency factor. The model presented above contains a fine-structure intermittency factor, γ^*, similar to the model by Corrsin (1962). He suggested that the small-scale structure consisted of vortex sheets occupying a volume fraction proportional to η/L, where L is an integral scale. This fraction is proportional to $R_\lambda^{-3/2}$. Modifying this, Tennekes (1968) proposed that the small scales were vortex tubes with a volume fraction η^2/λ^2. This quantity is proportional to R_λ^{-1}, that is, a larger fraction than in Corrsin's model. In their model, Frisch *et al.* (1978) suggested that eddies of size ℓ_n in the inertial range occupy a volume fraction $(\ell_n/L)^{3-D}$. From theory, they found that 2.5 was a likely value for the constant D, while Lesieur (1990) suggested a value closer to 3, based on experimental data. If this model was applied to the Kolmogorov scales, the volume fraction would have been larger that in the models by Corrsin and Tennekes.

Kolmogorov (1962) and Frisch *et al.* (1978) have suggested correcting the 5/3-law (Eq. 16) for intermittency in the inertial subrange. According to Lesieur (1990:158) these corrections are too small to allow experimental verification.

Experiments by Kuo and Corrsin (1971, 1972) indicated an intermittent structure somewhere between the sheets of Corrsin and the tubes of Tennekes, namely a "strip" or "ribbon"-like geometry. Recent experiments by Dahm *et al.* (1991) and Bish and Dahm (1995) showed a sheet-like distribution of the scalar dissipation rate in turbulent flows with Schmidt numbers $Sc \geq 1$, with and without combustion.

Direct numerical simulations are often truncated at a wavenumber of order η^{-1} and therefore give limited information about the fine structures. Some simulations have been done with much larger wavenumbers, but these still have a relatively low Reynolds number. Moreover, simple phenomenological models like sheets or tubes are not likely to depict the instantaneous details of the complex turbulence structure. However, such models are useful for understanding and simulating effects of turbulent flow.

The question remains what the fine structures actually are. The Kolmogorov microscale Reynolds number is unity by definition. These characteristic scales represent volume averages of the fine structures. The local velocity might be of the same order as the root-mean-square turbulence velocity, that is, orders of magnitude larger than the Kolmogorov velocity scale. Following the ideas of Richardson (1922) eddies are likely to break down to smaller scales, until inertia forces are locally balanced by molecular forces. From this reasoning, small scales must exist with wavenumbers that are considerably larger than η^{-1}. The lifetime and total volume of these scales is small, though, and their contribution to the total kinetic energy and dissipation may therefore be small.

5.5 Decaying Turbulence and Cascade Model

Modeling of decaying turbulence was discussed above. The viscous terms are important for the transition from strong to weak turbulence. However, the question arises whether weak decaying turbulence has relevance to the energy cascade. When the turbulence is weak, q' has the same magnitude as w'', and then the cascade reduces to a single level. The limit can be drawn where $2\omega' = \omega'' = \omega^*$, that is, when the cascade has two levels. Then, $w'' = w^* = q^* = 4q'$. With $C_{D1} = 0.135$ and $C_{D2} = 0.5$, this gives $R_\lambda = 24$ in Eq. (32). In other words, it is not certain that the limitations for the range of C_{D2} values that fit the decaying turbulence are relevant for the cascade model.

On the other hand, as discussed in the preceding section, it is likely that the flow has smaller structures than the fine structures. That is, there is a transfer of energy to higher frequencies than those of the fine structures in the spectrum. The mechanisms for this energy transfer and dissipation are believed to be the same as at lower frequencies. Therefore, we believe that the model $\varepsilon = w'' + q'$ is applicable to very low turbulence Reynolds numbers as well, and that the conceptual formulation should be the same.

5.6 High and Low Reynolds Numbers

In Section 3.2, the model has been compared to data and theory for the inertial subrange of the energy spectrum. This range occurs at high Reynolds numbers. At lower Reynolds numbers, the shape of the spectrum changes and there is no inertial subrange, and the applicability of Kolmogorov's 5/3-law is reduced.

Combusting flows often have relatively low turbulence Reynolds numbers, due to increased viscosity and reduced density at higher temperatures. Typically, the kinematic viscosity can be increased by a factor 10-30 in the reaction zone. As the length scales and velocity scales are not necessarily changed correspondingly, the limits for "low" and "high" Reynolds number have been changed.

The Eddy Dissipation Concept is based on one-point turbulence modeling. Thus, there is no information about the shape of the spectrum. This information can only be found from two-point closures. A cascade model provides a prescribed shape of the spectrum. When the turbulence energy is calculated, the area of the spectrum is given, and when the turbulence Reynolds number is calculated from turbulence energy and length scale (or dissipation), the width of the spectrum is given.

It has also been found that the high-Reynolds-number spectral data for nonreacting flows indicated a larger value for the model constant C_{D2}, while reacting flow spectral data and the decaying turbulence at low Reynolds numbers indicated a lower value. We think this reflects the situation where a one-point closure has to be a compromise between different regimes in turbulent flows, and that the set of constants in the model is a reasonable choice that represent flows at different Reynolds numbers. Moreover, combustion seems to change the energy spectrum.

5.7 Reactions and fine structures

Combustion takes place were reactant molecules are mixed and where hot products are mixed with reactants. In the presented model, this is related to the dissipation of turbulence energy. The dissipation is spread over a wide range of wavelengths or frequencies but occurs mainly in the small-scale structures. Here, the local viscous stresses are the largest. The Kolmogorov scale, η, is regarded as a representative scale for these motions. Similarly, the mixing mainly takes place in structures characterized by the Batchelor scale, $\eta/\sqrt{Sc}$. For the major species of combustion, and most minor species, the molecular Schmidt number Sc is close to unity. That is, the mixing and dissipation occur in the same structures. Motions at scales smaller than the Kolmogorov scales seem to be dependent on the Kolmogorov scales (see e.g. Zhou, 1993). As has been shown in Sec. 3.2, the presented model is in reasonable agreement with available data for the energy and dissipation spectra.

A further issue is the relevance of isothermal energy spectrum data to combustion. As mentioned, only few spectrum data for combusting flows are available. The effect of combustion is both to increase and to decrease the small scale turbulence. Locally and instantaneously the motions due to expanding hot gases will enhance the turbulence. On the other hand, larger local viscosity increases the local dissipation. The referred experiments seem to indicate that when averaged continously (as in one-point moment modeling) combustion has only minor effect on the energy spectrum.

6 CONCLUDING REMARKS

The turbulence energy cascade model that formed the basis for the Eddy Dissipation Concept for turbulent combustion has been presented. The cascade model consists of a stepwise model for energy transfer from larger to smaller scales. This gave a connection between the fine structures, where the main part of the molecular mixing takes place, and the characteristics of the larger eddies, which generally are modeled by turbulence models like the k-ε model or similar. The various terms entering the combustion model have been presented and discussed.

The large-eddy, or inertia-dominated, part of the cascade model was related to turbulence viscosity of two-equation models, and to experimental data from the logarithmic boundary layer. The model was also related to the inertial subrange of the energy spectrum and corresponding data for different types of flows. In this domain, the model was found to be consistent with data and existing models. The fine-structure part of the cascade, where viscous forces are important, was related to models and data for the dissipative range of the energy spectrum, for small-scale intermittency, and for the late stages of isotropic decaying turbulence. To the extent that one-point statistical models can supply input turbulence quantities to the cascade model, the presented model was found to reproduce such flows.

It is concluded that the energy cascade model, which is the basis of the Eddy Dissipation Concept for combustion, captures important features of reacting and nonreacting turbulent flows. The expressions and constants are compromises that aim at a wide validity domain.

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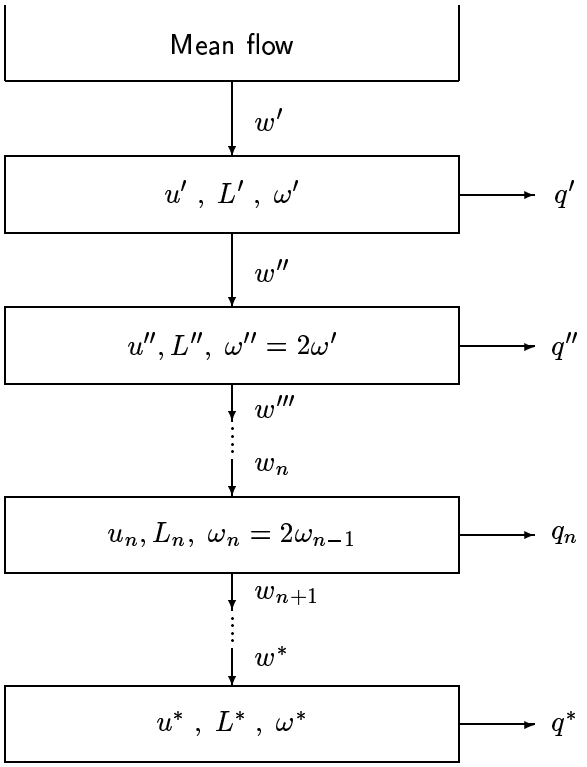

Figure 1: Energy cascade model for the transfer of mechanical energy from the mean flow, through turbulence energy to heat.

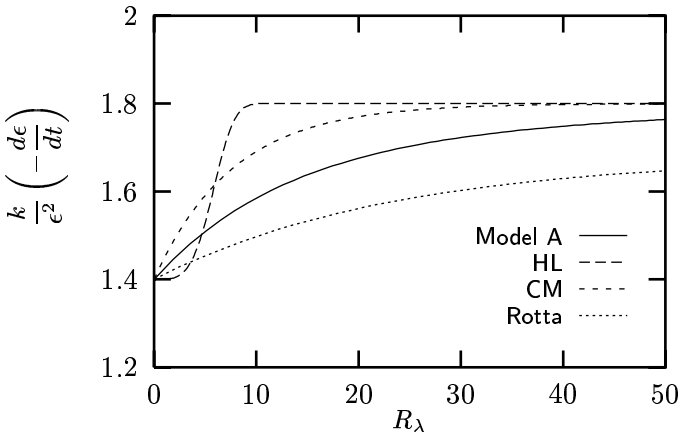

Figure 2: Nondimensional decay of dissipation rate for the present model compared with the models of Rotta (1951a), Hanjalić and Launder (1976) (HL), and Coleman and Mansour (1991) (CM).

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