Department of Energy and Process Engineering ISE Jan.17
TEP4170 Heat and combustion technology

Exercise 2: Turbulence – averaging and modeling

Reynolds decomposition and averaging

Problem 1:

Use

$$\overline{\varphi} = \frac{1}{T}\int_{t_o-\frac{1}{2}T}^{t_o+\frac{1}{2}T} \varphi(t)dt \tag{1}$$

to show that

$$\overline{\overline{\phi}} = \overline{\phi} \tag{2}$$

$$\overline{\phi'} = 0 \tag{3}$$

$$\overline{\phi + \psi} = \overline{\phi} + \overline{\psi} \tag{4}$$

$$\overline{\overline{\phi} \cdot \overline{\psi}} = \overline{\phi} \cdot \overline{\psi} \tag{5}$$

$$\overline{\overline{\psi}\, \phi'} = \overline{\psi} \cdot \overline{\phi'} = 0 \tag{6}$$

$$\overline{\overline{\phi} \cdot \psi} = \overline{\phi} \cdot \overline{\psi} \tag{7}$$

$$\overline{\phi\, \psi} = \overline{\phi} \cdot \overline{\psi} + \overline{\phi'\, \psi'} \tag{8}$$

$$\overline{\frac{d\phi}{ds}} = \frac{d\overline{\phi}}{ds} \tag{9}$$

$$\overline{\int \phi ds} = \int \overline{\phi} ds \tag{10}$$

$$\overline{\left(\frac{\overline{\phi}}{\overline{\psi}}\right)} = \overline{\phi} \cdot \overline{\left(\frac{1}{\overline{\psi}}\right)} = \frac{\overline{\phi}}{\overline{\psi}} \tag{11}$$

Turbulence equations

Problem 2:

– Introduce the Reynolds decomposition in the basic equations and develop the following equations (assume that density does not vary):

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{12}$$

$$\frac{\partial u'_j}{\partial x_j} = 0 \tag{13}$$

$$\frac{\partial}{\partial t}(\rho \overline{u}_i) + \frac{\partial}{\partial x_j}(\rho \overline{u}_i \overline{u}_j) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\overline{\tau}_{ij} - \rho \overline{u'_i u'_j}\right) \tag{14}$$

– Why do we do this?
– What was lost during this operation? (besides the possible variation of denisty, which was neglected)

A turbulence model

Problem 3:

In Section 2.6 of the textbook (Ertesvåg), Prandtl's mixing length model is developed for momentum transfer. This is a model for the turbulence stresses in Eq. 2.8.

– Develop a similar model for the mass flux and the heat flux (see the second last terms of Eqs. 2.9 and 2.10). That is, determine the turbulence diffusivities $\mathcal{D}_\mathrm{t}$ and α_t in Eqs. 5.1 and 5.2 similar to the turbulence viscosity in Eq. 2.17.