Exercise 8: Energy spectrum, simple estimates

Problem 1:
a)
Starting from the following expression for dissipation of turbulence energy,

$$\varepsilon = A\frac{u'^{\,3}}{\ell} = 15\nu\frac{u'^{\,2}}{\lambda_g^2} = \nu\frac{v^2}{\eta^2} \tag{1}$$

develop the expressions

$$Re_\lambda^2 = \frac{15}{A}\,Re_\ell \tag{2}$$

$$\frac{\eta}{\ell} = A^{-\frac{1}{4}}\,Re_\ell^{-\frac{3}{4}} \tag{3}$$

$$\frac{\eta}{\lambda} = 15^{-\frac{1}{4}}\,Re_\lambda^{-\frac{1}{2}} \tag{4}$$

$$\frac{v}{u'} = 15^{\frac{1}{4}}\,Re_\lambda^{-\frac{1}{2}} \tag{5}$$

b)
Let ℓ be defined by $\nu_\mathrm{t} = u'\ell$ and determine A by comparing with the k-ε model.

Problem 2:

Assume that the energy spectrum in isotropic turbulence is expressed as

$$E(\kappa) = \begin{cases} A\kappa^m & \text{for} \quad \kappa \le \kappa_L \\ \alpha\varepsilon^{\frac{2}{3}}\kappa^{-\frac{5}{3}} & \text{for} \quad \kappa \ge \kappa_L \end{cases}$$

a) Determine the turbulence energy as a function of time.
b) How will this function bee for $m = 1$, $m = 2$ and $m = 4$?

Problem 3:

A "cloud" of light particles are introduced into a steady flow of isotropic turbulence. the initial diameter of the cloud is $D_\circ$, where $\eta \ll D_\circ \ll L_g$.
(η is the Kolmogorov scale, L_g is an integral scale).

Find how the diameter D of the cloud varies while $D \ll L_g$.

Problem 4:

Gas flows in a straight duct (channel) with a square cross-section. The width of the duct is 0,1 m, and the volumetric flow is 0,1 m^3/s. The kinematic viscosity can be assumed at $\nu = 1 \cdot 10^{-5}\,\mathrm{m}^2$/s.
– Verify that the flow is turbulent
– Estimate the turbulence energy (k), its dissipation rate (ε), the Kolmogorov length scale (η) and time scale (τ).
– Estimate the turbulence viscosity ν_t.
– Estimate the necessary frequency resolution for equipment to measure turbulence parameters.
Reactions occur in the flow, and the relevant reactions have time scales from 10^{-6} s to 10^{-2} s.
– Estimate the maximum timestep of a numerical simulation with full resolution (no modelling) of the reacting flow.